An Analysis of Broadband Circulators with External Tuning Elements

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Abstract—**Equations are presented which show how the isolation-bandwidth characteristic of a given Y-circulator junction can be modified by external tuning elements. Results are given for tuning with one, two and, as a limiting case, an infinite number of resonant elements. The results suggest that there is only limited room for improvement in the best of the current state-of-the-art empirically designed circulators.**

INTRODUCTION

FROM THE BEGINNING, microwave engineers have realized that the experimental design of circulators is a matching problem, and have relied on the theorem that any lossless 3-port junction that is matched at all ports must be a circulator. The design of the circulator junction itself usually proceeds on a cut and try basis until circulation is obtained at the desired center frequency. Careful measurements of the input admittance as a function of frequency can then serve as the basis for the design of external tuning elements to broadband the device. In this paper, some simple approximate analytical techniques are presented for establishing a rational starting point for the broadbanding procedure, and for estimating what the fundamental limits in performance are likely to be. The results will be restricted to the 3-port Y-junction circulator.

EQUIVALENT CIRCUIT OF THE CIRCULATOR JUNCTION

Recently, substantial progress has been made in formulating a theory of circulator operation based on resonant electromagnetic modes of the junction itself. One can cite here particularly the work of Bosma [1], Butterweck [2], and Fay and Comstock [3]. The theory is especially useful because the formalism can be made the basis of an analytical design procedure, even when the junction geometry is so complicated as to preclude even an approximate solution to its boundary value problem. The technique is particularly suited to circulators which need operate over only moderate bandwidths (typically 20 percent), but must have within that band very precisely specified electrical properties. This is often the case with circulators for high performance parametric amplifiers.

According to Fay and Comstock [3], an operating 3-port circulator, viewed from the input port, has the equivalent circuit shown in Fig. 1, subject to the auxiliary conditions

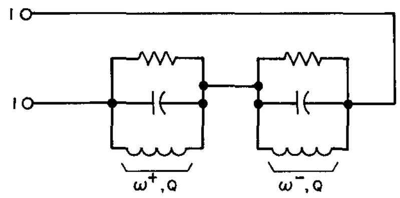

Fig. 1. Equivalent circuit looking in the input port of a 3-port Y-circulator adjusted for circulator action. At midband the susceptances of the two resonators are equal, but of opposite sign, and cancel, leaving a real, matched input resistance.

$$\omega_0 = \frac{\omega^+ + \omega^-}{2} \tag{1}$$

$$\Delta\omega \equiv \omega^+ - \omega^- = \frac{\omega_0}{\sqrt{3}\,Q}, \tag{2}$$

where ω_0 is the center of the operating band, ω^+ and ω^- are the resonant frequencies of the dipolar modes of the central disk [2], [3] and Q is the external Q of each mode when it is loaded by the output port.

The extent to which this simple circuit can characterize a real circulator in the immediate vicinity of the junction resonance is shown in Fig. 2, which shows the admittance, as a function of frequency, of a symmetrical 3-port L-band circulator. The coupling in this case was quite loose, corresponding to a high external Q, since the junction was connected directly to 50 ohm transmission lines. The dots represent careful measurements [4] made at equal frequency intervals; the solid curve was computed from the equivalent circuit of Fig. 1, with the Q chosen to match the band-edge susceptance. Perfect agreement would require that the pips and dots coincide. It is evident that the measured susceptance is in good agreement with the model, the conductance less so.

Because the junction used for the above measurements had a high external Q, meaningful admittance measurements were restricted to a relatively narrow frequency range (~12 percent). Measurements over a larger frequency range would be possible with tighter coupling, by means of a quarter-wave transformer for example, but at the expense of uncertainty in referring the measured admittance to the junction itself. As a result of these experimental uncertainties, the use of the relatively complex two-resonator equivalent circuit cannot really be justified over a wide frequency range. Instead, we shall simply approximate the junction by a single shunt resonator whose admittance consists of a constant conductance G_R and a variable susceptance characterized by

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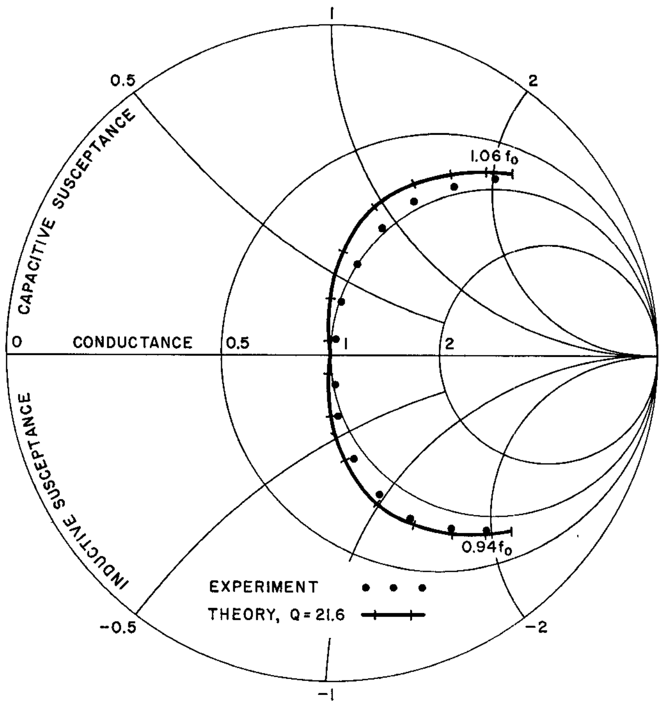

Fig. 2. Comparison of theory and experiment for the input admittance of a loosely coupled L-band circulator.

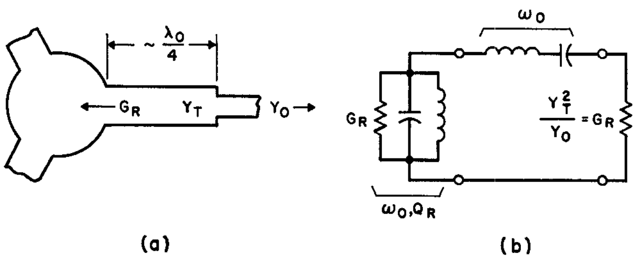

Fig. 3. Center conductor geometry and lumped element equivalent circuit for a quarter-wave transformer coupled circulator.

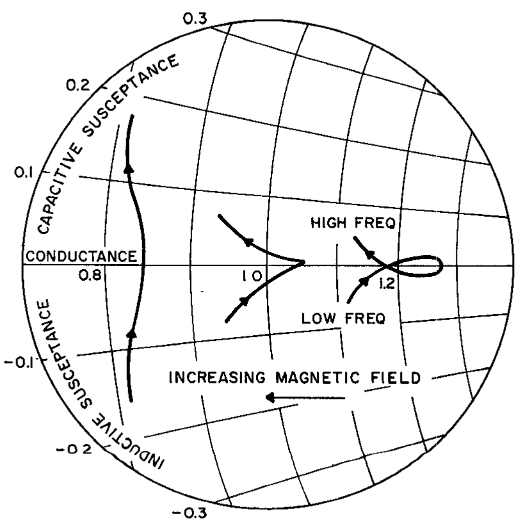

Fig. 4. Measured input admittance of a quarter-wave transformer coupled Y-junction circulator at different magnetic fields, showing the increasing tendency to approximate a constant resistance.

a loaded Q, Q_R. One of the utilities of this approach is that just as in the conventional microwave cavity resonator [5], the product $Q_R G_R$ depends primarily on the geometry of the junction itself, and only secondarily on the details of how it is externally loaded. Thus broadband circulator design can be separated into two phases; choice of a basic junction geometry having a small value of $Q_R G_R$, and then the design of a suitable coupling network. The balance of this paper is concerned only with the second part of the problem, i.e., in effect, with the broadband matching of a simple shunt resonator. In this respect, the approach is very similar to that used by Konishi [6], who directs his attention specifically to lower frequency lumped element circulators. The reader interested in a general treatment of the problem of broadband matching to reactive loads is referred to standard texts [7], [8].

Quarter-Wave Transformer Matching

The quarter-wave transformer is basic to any practical matching scheme for stripline circulators [3]. A typical center-conductor configuration is shown in Fig. 3(a), and an equivalent circuit is shown in Fig. 3(b). In keeping with the approximation introduced above, the junction is represented by a shunt resonator. Near band center the frequency dependence of the electrical length of the quarter-wave transformer can be represented by the series resonator shown. Thus between the junction and the transformed load we have in effect a half-section bandpass filter, the characteristics of which can be chosen to get the desired performance. An analysis of this problem is presented in the Appendix, where approximate design relationships are derived for the transformer characteristic admittance and junction Q. If the VSWR is allowed to ripple in the passband, with a maximum value ρ_M at midband and at the band edges, the required junction Q is given by

$$Q_R \approx 1.4 \frac{(\rho_M - 1)^{1/2}}{\Delta f/f_0}, \tag{3}$$

where $\Delta f/f_0$ is the normalized bandwidth. The required transformer admittance Y_T is then given approximately by

$$\frac{Y_T}{Y_0} \approx \rho_M \left[1 + \frac{4Q_R}{\pi}\right]^{1/2}, \tag{4}$$

where Y_0 is the characteristic admittance of the output stripline. Typical performance which has been achieved by this technique is a 35 dB isolation bandwidth of 13 percent, or, alternatively with the same basic junction, 30 dB at 20 percent.

Series and Shunt Tuned Circulators

By increasing the junction conductance over what one would use in a normal quarter-wave transformer coupled circulator, and adding another tuning element, better performance can be obtained [9], [10]. This concept is traced through in Fig. 4, which shows the input admittance of a below-resonance circulator, measured at the outer end of the quarter-wave transformer, for three values of the dc field on the junction. As the magnetic field is increased, which has the effect of increasing the junction conductance, the characteristic opens up, in addition to shifting along the conductance axis. At some particular value of magnetic field the characteristic will approximate that of a series resonant circuit, but with a negative L and C. In the example shown the point of best approximation to a negative series resonator occurs for a normalized midband conductance less than unity, but with a somewhat different choice of field and transformer this can be made to occur at the center of the chart. If a simple series resonant circuit is now added at the end of the transformer, the locus of the input admittance collapses almost to a single point.

A practical technique [9], [10] for realizing this behavior is shown in Fig. 5(a). Here, in addition to the quarter-wave transformer, a quarter-wave open-circuited stub has been added in series with the center conductor. Any direct coupling of this stub to the ground planes must be kept small, of course, if it is to be effective as a series element. Figure 5(b) shows the arrangement more schematically, while the lumped equivalent circuit referred to the junction is shown in Fig. 5(c). As before, the junction is represented by a shunt resonator, while the series resonator accounts for the frequency dependence of the transformer. The stub itself looks like a series resonator, but when transformed back through the quarter-wave section it becomes a shunt resonant circuit. Thus, between junction and load there is now a full π-section bandpass filter. The exact choice of element values depends in detail on the kind of admittance characteristic desired. For example, if we arbitrarily decide to bring the band-edge points as well as midband to a perfect match, an analysis similar to that summarized in the Appendix for the case of simple quarter-wave transformer matching yields the following approximate design equations:

$$Q_R \approx 1.1 \frac{(\rho_M - 1)^{1/3}}{\Delta f/f_0} \tag{5}$$

$$\frac{Y_T}{Y_0} \approx \frac{8Q_R}{\pi} \tag{6}$$

$$\frac{Y_s}{Y_0} = \frac{\pi}{4Q_R}, \tag{7}$$

where Y_s is the characteristic admittance of the series stub, and ρ_M is again the maximum VSWR in the passband.

Theory and experiment for a series-tuned circulator are compared in Fig. 6. The dotted curve shows the input admittance measured over a 40 percent bandwidth on an empirically designed L-band unit [9]. The solid curve is a computed characteristic, obtained by starting with the measured junction Q and applying the preceding design procedure to get the remaining element values. If we examine the inset, we find that theory would predict a 16 percent band within the 1.02 VSWR circle. With a small, primarily susceptive correction, the middle 16 percent of the experimental curve could also be shifted within the 1.02 VSWR circle. Out of band, the measured and computed characteristics are substantially different, reflecting primarily the inadequacy of the simple equivalent circuits used. In band, however, they are close enough to give one confidence that the element values obtained from the design procedure represent a reasonable jumping-off point for further experiment.

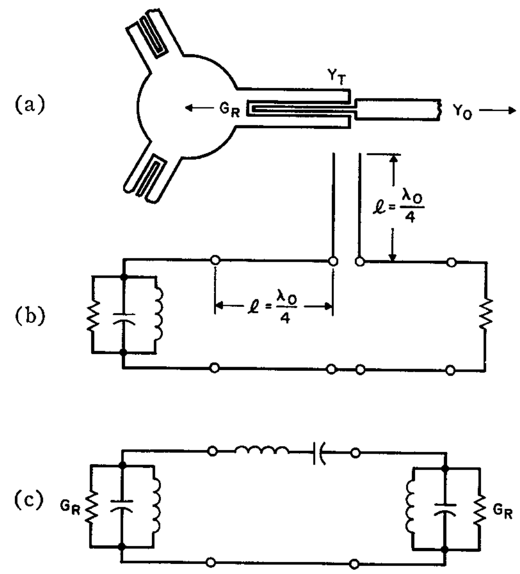

Fig. 5. Center conductor geometry and lumped element equivalent circuit for a series-tuned circulator.

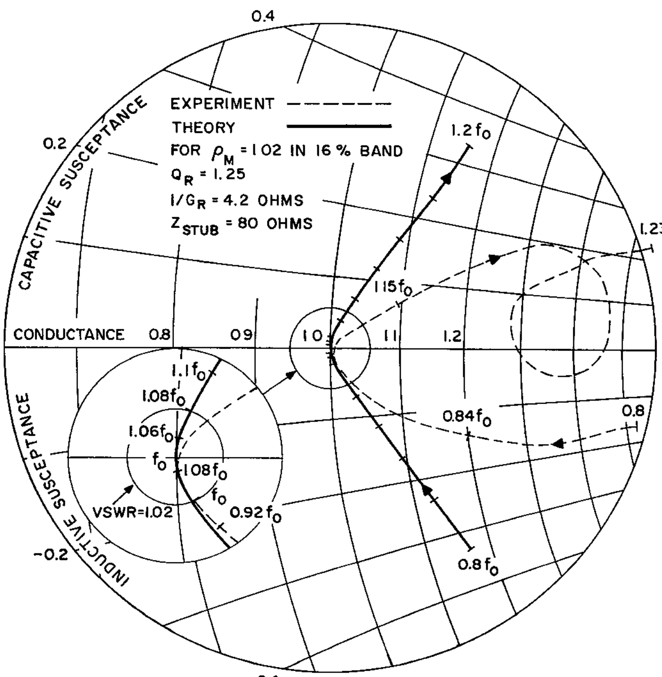

Fig. 6. Measured and computed input admittance for a series-tuned circulator. The inset is a blown-up view of the center of the Smith chart.

The junction Q which was used in the empirically designed unit was not optimum, in the sense that greater bandwidth could have been achieved for the same junction Q, or the same performance could have been realized with a higher junction Q. For example, the design procedure predicts that a VSWR of 1.02 could be achieved in a 20 percent band by a Q as high as 1.45, compared with 1.25 for the experimental unit. The required performance is achieved in

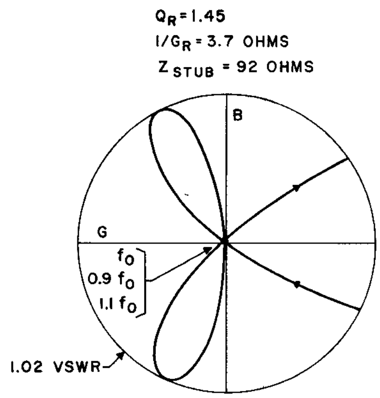

Fig. 7. Computed input admittance of a series-tuned circulator designed to have 40 dB isolation in a 20 percent band.

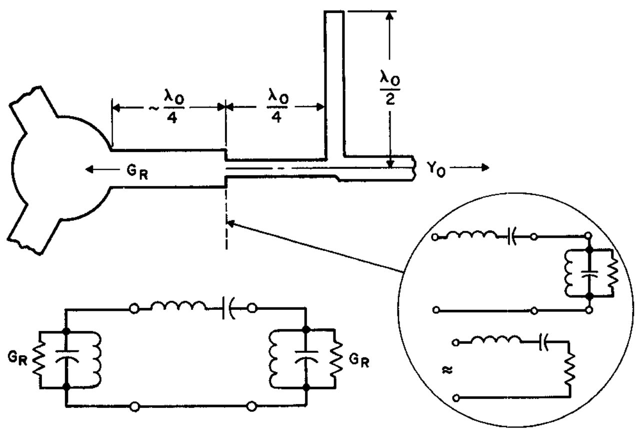

Fig. 8. Center conductor geometry and lumped element equivalent circuit for a shunt-tuned circulator.

spite of the higher Q by allowing the characteristic to loop, in band, as shown in Fig. 7. The grouping here, in fact, is not the tightest that could be achieved, but reflects the arbitrary choice, made in arriving at the design equations, to bring the band-edge points as well as midband to a perfect match.

This so-called "point match" performance can also be achieved using shunt tuning elements [4], rather than the series elements previously discussed. One center conductor arrangement for achieving this is shown schematically in Fig. 8. Besides the basic transformer-coupled junction it consists of a half-wavelength open-circuited stub and an additional quarter-wavelength transformer section. It can be shown, by transforming all elements back to the junction, that near band center the equivalent circuit for this arrangement is identical to that of the simple series-tuned circulator. The steps in this transformation are shown in the inset. Viewed from the reference plane shown, the second quarter-wave transformer looks like a series resonator followed by a frequency-independent quarter-wave transformer. This latter transformer inverts the shunt resonator formed by the shunt stub, so that there are, in effect, two series resonators in series. These combine to form a single series resonator, just as in the case of the series-tuned circulator. We thus expect comparable performance. The main advantage of the shunt technique is the increased flexibility in the choice of junction impedance level that the additional quarter-wave transformer allows. The shunt arrangement also has some practical advantages; it is more suited to printed circuit techniques, and the tuning elements are accessible for trimming adjustments. The series technique, on the other hand, provides a built-in dc block in the center conductor which can simplify the problem of getting bias on the diode in some parametric amplifier applications.

Typical performance which has been realized with the added tuning is 30 dB isolation ($\rho_M = 1.065$) in a 30 percent band, corresponding, according to (5), with a junction Q of 1.4.

Fundamental Limits on External Matching

Comparison of (3) and (5) shows clearly that for a given junction greater bandwidth or improved performance can be obtained by increasing the number of external tuning elements, and indicates the trade-offs involved. The obvious question to ask at this point is just how far this can be carried. Elementary network theory can provide some indication of fundamental limits. There exists, for example, a theorem due to Bode [8] which establishes the maximum return loss which can be realized in a given band in the presence of "parasitic" shunt reactance. Mathematically, Bode's result can be written

$$\log_2 \left| \frac{1}{\Gamma} \right| \leq \frac{\pi}{(\omega_2 - \omega_1)RC}, \tag{8}$$

where Γ is the voltage reflection coefficient, C the capacitance shunting a source of resistance R, and $\omega_2 - \omega_1$ is the frequency interval of interest. The equality can hold only if the reflection coefficient has a constant value in the passband and magnitude unity elsewhere. We can apply this result to a circulator junction by accounting for its shunt susceptance in terms of a Q, as was done before, and setting $RC = Q_R/\omega_0$. By expressing the reflection coefficient in terms of the return loss, in dB, which is also equal to the isolation, we obtain

$$Q_R \frac{\Delta f}{f_0} \leq \frac{27.2}{\text{Return Loss}} \tag{9}$$

where $\Delta f/f_0$ is the normalized passband and the return loss is in dB.

Conclusions

The preceding results show that once the basic junction Q and bandwidth have been determined, there is a definite upper limit to the return loss, or isolation, that can be realized in the passband, regardless of the sophistication of the matching circuitry. It is interesting to compare this upper limit with the results that can be achieved with the simple quarter-wave matching transformer, or the combination of transformer and series element. For these latter cases (3) and (5) show that the product $Q_R\ \Delta f/f_0$ uniquely determines the maximum achievable in-band return loss or, equivalently, the minimum VSWR in the passband. These relations are portrayed graphically in Fig. 9. We have included simple transformer coupling, the transformer plus series resonator as well as the simple direct coupled circulator, in which the

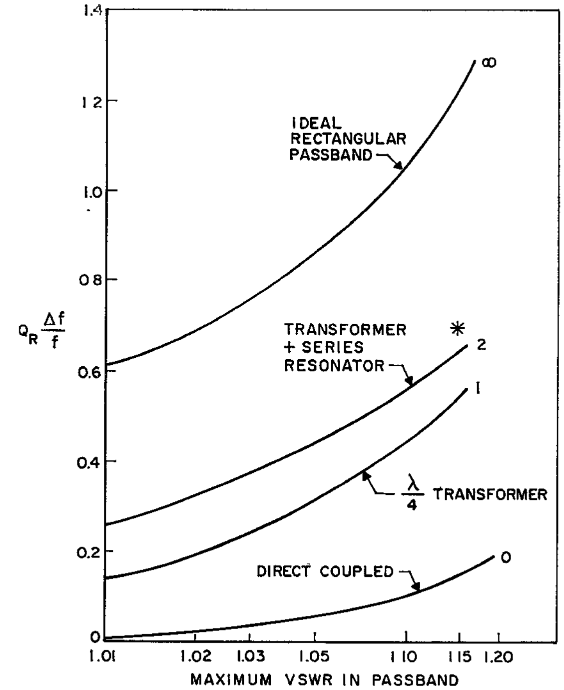

Fig. 9. The product of junction Q and normalized bandwidth as a function of the maximum VSWR in the passband. The variable parameter is the effective number of resonant tuning elements in the external matching network.

junction is connected directly to the outside world with no tuning whatever. The numbers at the right refer to the number of effective resonant tuning elements. The upper curve is the limiting case given by (9), and would require, as indicated, an infinite number of elements.

These curves suggest a number of interesting observations. For example, it is relatively more and more advantageous to increase the number of tuning elements as one demands higher performance from the device. Thus, at a VSWR of 1.02, the series-tuned circulator offers a 65 percent increase in bandwidth over the simple quarter-wave transformer coupled circulator, while the ideally matched circulator would be better by a factor of about 3.5. If, however, the VSWR can go to 1.15, series tuning offers an improvement of only 14 percent, and ideal matching a factor of 2.2.

It is also interesting to speculate on what the ultimate performance of an octave bandwidth circulator might be. Measurements made at the lower microwave frequencies have suggested that junction Q's as low as unity might be realized. The upper curve indicates that ideally we could then obtain an octave bandwidth (corresponding to an ordinate of 0.67) with a VSWR of less than 1.02. The current state-of-the-art octave bandwidth circulators have VSWR's of perhaps 1.15; this point is marked with an asterisk on Fig. 9. It might be possible, by adding more tuning elements, to improve the performance or increase the bandwidth (with roughly equal difficulty) but this will clearly not be easy, since current circulators are evidently already in a region of diminishing returns, where the addition of another tuning element has only a small effect.

Appendix

Derivation of Design Equations for Quarter-Wave Transformer Coupled Circulators

With reference to Fig. 3(a), the normalized input admittance at the output end of the quarter-wave transformer is

$$y \equiv \frac{Y}{Y_0} = \frac{Y_T}{Y_0}\frac{Y}{Y_T} = \frac{Y_T}{Y_0}\left(\frac{Y_T - jY_L \cot \beta l}{-jY_T \cot \beta l + Y_L}\right), \quad (10)$$

where $Y_L \approx G_R(1+j2Q_R\delta)$ is the admittance of the circulator junction, and $\beta l = (\pi/2)(1+\delta)$ is the electrical length of the transformer section, $\delta \equiv (\omega-\omega_0)/\omega_0$ being a normalized frequency parameter. Equation (10) can be expanded in powers of this frequency parameter. Correct to terms of order δ^2, the result is

$$y \approx K\left\{\frac{1+\frac{\pi^2}{4}\delta^2 + j\frac{\pi}{2}\left[\left(n-\frac{1}{n}\right)-\frac{4Q_R}{\pi}\right]\delta}{1+\left(\frac{\pi}{2n}+2Q_R\right)^2\delta^2}\right\}, \quad (11)$$

where we have introduced the abbreviations

$$K = \frac{{Y_T}^2}{Y_0 G_R} \quad \text{and} \quad n = \frac{G_R}{Y_T}.$$

The detailed shape of the admittance locus (i.e., whether it loops or cusps) actually depends on terms in δ^3, but in view of the approximation already made of representing the junction by a shunt resonator, it is pointless to carry the calculation that far. For our purposes it is enough to observe that the optimum choice of transformer and junction Q will never be far from the values that cause the linear susceptance term in (11) to vanish. This gives us one design equation,

$$n - \frac{1}{n} = 4\frac{Q_R}{\pi}. \quad (12)$$

With this choice, the admittance, in the approximation used here, is real,

$$y = g = K\left[\frac{1+\frac{\pi^2}{4}\delta^2}{1+\left(\frac{\pi}{2n}+2Q_R\right)^2\delta^2}\right]. \quad (13)$$

The other design equation arises from a specification of the maximum allowable VSWR, ρ_M. Maximum bandwidth can be obtained by allowing VSWR ripples in the passband, with equal VSWR's at midband and the band edges. In terms of the circulator admittance, this means that

$$g_{\text{high}} = g_{\text{low}} = \frac{1}{g_{\text{mid}}} = \frac{1}{\rho_M}, \quad (14)$$

Equations (13) and (14) together yield a second design equation,

$$\frac{1}{\rho_M^2} = \frac{1 + \left(\frac{\pi}{4}\right)^2 \left(\frac{\Delta f}{f_0}\right)^2}{1 + \frac{1}{4}\left(\frac{\pi}{2n} + 2Q_R\right)^2 \left(\frac{\Delta f}{f_0}\right)^2}, \tag{15}$$

where Δf is the bandwidth.

Equations (12) and (15) can be solved simultaneously to yield expressions for the required transformer ratio n and junction Q, Q_R. These are

$$\left(\frac{\pi}{4Q_R}\right)^2 = \frac{1}{(1+b^2)^2}\left(1 + \frac{\rho_M^2 b^2}{\rho_M^2 - 1}\right)\left(\frac{b^2}{\rho_M^2 - 1}\right), \tag{16}$$

and

$$n^2 = \frac{\rho_M^2(1+b^2) - 1}{b^2}, \tag{17}$$

where $b = (\pi/4)(\Delta f/f_0)$ is a convenient bandwidth parameter. These expressions are somewhat unwieldy. Considerable simplification results, however, if we recognize that for the circulators of interest here ρ_M is close to unity, so that $\rho_M^2 \approx 1 + 2(\rho_M - 1)$, and $b^2 \ll 1$. Also, for the designs of interest, the quantity $b^2/(\rho_M^2 - 1)$ is substantially less than unity. Thus, to within an accuracy of 10 to 20 percent, (16) and (17) can be written simply as

$$\left(\frac{\pi}{4Q_R}\right)^2 = \frac{b^2}{2(\rho_M - 1)} \tag{18}$$

and

$$n^2 = 1 + \frac{2(\rho_M - 1)}{b^2} \cdot \tag{19}$$

Equation (18) leads directly to (3) of the text. Equation (4) of the text is obtained from (19) by noting that $Y_T/Y_0 = Kn = \rho_M n$.

Acknowledgment

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